A Control Theory of Financial Structure: Outside Equity Control and the Priority and Maturity Structure of Debt*

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Abstract

Firms' financial structures typically consist of debt claims of different priority and maturity, and outside equity with control rights. The present paper develops a simple control theory of financial structure in which these features arise endogeneously to allocate control and cash flow rights among the firm's manager and its investors. While short-term debt commits the manager to liquidate the firm in low profit states, outside equity with unconditional control allows investors to seize control in states for which the manager otherwise would pursue low profit projects that yield high private benefits of control. Finally, long-term subordinated debt protects the manager from excessive shareholder involvement.

1 Introduction

The financial structures of firms typically consist of debt claims of different priority and maturity, and outside equity with unconditional control. This paper presents a simple control argument in which these features arise endogenously to allocate control and cash flow rights among the firm's claimants. In particular, we examine a setting in which the presence of private benefits of control creates a divergence between the actions desired by the firm's manager and those desired by its investors. Financial structure in this setting arises as a tool to implement the optimal (ex ante) contract between the firm's manager and its investors. The type of financial structure needed for this consists of short-term senior debt, long-term subordinated debt, and outside equity.

Short-term debt commits the manager to transfer control to the firm's investors in low profit states for which it is optimal to liquidate the firm.[1] Outside equity with unconditional control allows security holders to implement projects that generate high returns but low private benefits to the manager (and hence are resisted by the manager). However, if the financial structure of the firm consists of short-term debt and outside equity only, then investors will be able to liquidate the firm, or to replace its manager, whenever this is consistent with value maximization, regardless of the optimal contract ex ante. Hence, there is a need to soften the incentives of shareholders to intervene on the interim date, which is done by issuing issuing long-term debt. This creates a debt overhang on the interim date, which forces shareholders to share any value improvements with the long-term debt holder and in turn reduces their incentive to intervene.[2]

Many of the results on debt maturity and priority structure generated are analogous to those derived by Diamond (1993), although the basic assumptions differ. For example, while Diamond considers asymmetric information, we consider symmetric information. (However, both arguments are control driven with optimal control transfers based on unverifiable interim information). Furthermore, Diamond ignores the role of outside equity to focus "on the effects of debt on transfer of control, thus avoiding takeovers as another way of transferring control." In the present paper outside equity *complements* debt in implementing the optimal contract.

Other (and related) control based theories on capital structure include Chang (1992),

[1] This is analogous to Harris and Raviv (1990), Chang (1992), Aghion and Bolton (1992), Diamond (1993), Dewatripont and Tirole (1994), and Hart and Moore (1989, 1994), among others.

[2] This defensive role of debt financing is analogous to Israel (1991), who shows that a higher level of debt will reduce the probability of a takeover while at the same time it will increase the value collected by the target's shareholders in the event of a sucessful bid. In the present model issuing *long-term* debt, as opposed to debt in general, is purely defensive and done in order to reduce the probability that shareholders will take actions to replace the current manager.

Berkovitch and Israel (1996), Berkovitch, Israel, and Spiegel (1998), Zwiebel (1996), Dewatripont and Tirole (1994), and Joskinen (2000).[3] With the exception of Joskinen (2000), none of these (including Diamond) produce the type of financial structure generated here where both capital structure (debt and equity) and debt structure (priority and maturity) matter. Also, our argument is more purely control driven than those found in these related papers, showing that the control argument alone (along with non-verifiable information) can generate the type of complex financial structures observed in practice.

Chang (1992) shows that short-term debt can be used to implement the optimal contract between the firm's investors and its manager, but ignores the role of outside equity and consequently the role of debt maturity and priority. In Dewatripont and Tirole (1994) the firm's capital structure (debt and equity) serves the dual role of inducing managerial effort and allocating control and cash flow rights. In the present paper, capital structure serves only the latter function. Although in both papers long-term debt serves to protect the manager's control, in their paper it protects the manager from too much involvement from the firm's short-term senior creditors, while in the present paper it protects the manager from too much involvement by shareholders.[4] Koskinen (2000) develops a model in which the firm's debt structure arises as a commitment device to prevent the outside shareholder from acquiring (too much) information about the manager's type.[5]

In Berkovitch and Israel (1996) and Berkovitch, Israel, and Spiegel (1998) capital structure serves the dual role of inducing effort by the manager and implementing an optimal replacement rule. In the present paper, capital structure serves only the latter role, but attains a richer structure by including a role for both debt maturity and priority structure. In Zwiebel (1996) issuing debt commits the manager not to undertake unprofitable projects. As such, the presence of debt protects the manager from being replaced and hence protects his control rent in the continutation project of the firm. In the present paper, (short-term) debt commits the manager to liquidate the firm, while it (long-term debt) also gives the manager some slack to pursue unprofitable (but high rent) projects. In both papers, outside equity with unconditional control allows shareholders to replace the manager in certain states.

In Berglöf and von Thadden (1994) the role of debt is to induce the manager-entrepreneur

[3] See Aghion and Bolton (1992) for a general formulation, and Hart (1995) for a review of some of the general issues involved.

[4] In their paper the amount of long-term debt is determined by the manager on the interim date and is contractually linked to the firm's first period profit level. Long-term debt can feasibly be used to implement the optimal policy (which is based on unverifiable information and hence cannot be contracted upon directly) in their setting since it is measurable with respect to the first period profit level (which is verifiable at no cost and hence can be linked to the amount of long-term debt that the manager will be allowed to issue).

[5] See Burkart, Gromb, and Panunzi (1997) for a related role of ownership structure.

to distribute cash to the firm's investors. As in the present paper, the firm's debt structure consists of long-term junior debt and short-term senior debt. In their setting, long-term debt strenghtens the bargaining power of the short-term lender, which in turn reduces the likelihood of the borrower defaulting strategically. In the present paper, neither strategic defaults nor inducing the manager to pay out cash to the firm's investors are relevant contracting frictions.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 derives the optimal contract between the manager and the firm's investors under the assumption of verifiable interim information. Section 4 examines the role of the firm's financial structure in implementing the optimal contract under unverfiable information. Section 5 derives the comparative statics of the model and relates these to the relevant empirical evidence. Finally, Section 6 concludes the paper.

2 The Model

The formal model has three dates—0, 1, and 2. A firm established on date 0, at which point its financial structure is determined and a manager is hired to run it. This manager will run the firm until at least date 1, and possibly until date 2. Everybody is risk neutral. The riskless interest rate is assumed to be zero.

A public information signal x is observed on date 1. This signal is non-verifiable, which implies that enforceable contracts cannot written directly on it (Grossman and Hart (1986)). Specifically, let x be a random variable with cumulative distribution function $F(x)$ and density $f(x)$; $f(x) > 0$ for all $x \in X = [\underline{x}, \overline{x}]$. The signal x provides information about the firm's date 2 cash flow and can be used to determine its date 1 value maximizing operating policy. Indeed, after observing x, the actions (or operating policies) available to the manager are *liquidate* (L), *continue* (C), and *restructure* (R). L and R are both viewed as costless changes in the firm's operating policy and hence require no additional investment outlays. C represents status quo and is the policy preferred by the manager.

Specifically, the date 2 cash flow under C is given by $x + \omega$, where ω has a zero mean with support $[\underline{\omega}, \overline{\omega}]$. Its distribution and density functions are denoted $G(\omega)$ and $g(\omega)$. It is assumed that $\text{cov}(\omega, x) = 0$, so that x represents the expected date 2 cash flow given C. In addition, C gives the manager control benefits of B (see e.g. Aghion and Bolton, 1992).[6] Alternatively, we may think of the cash flow generated by the firm as given by $x+\omega+B$, with $x+\omega$ appropriated by the the firm's security holders and B diverted by the manager.

[6]The presence of such private benefits of control are well recognized in the literature. Recently, Kaplan and Strömberg (1999) in examining venture capital contracts find that "the contracts we observe are most consistent with the [control based] theories of Aghion and Bolton (1992) and Dewatripont and Tirole (1994)."

Under L, the firm is liquidated on date 1, in which case the manager receives zero control benefits. The liquidation value of the firm is given by l, so that L represents value maximization relative to C for $x \in [\underline{x}, l)$. R generates a date 2 cash flow of $J(x) + \omega$, where $J'(x) > 1$ and $J(x) > x$ for $(x^*, \overline{x}]$, and where $x^* \geq \overline{x}$. Hence, R represents firm value maximization relative to C for any $x \geq x^*$. Note that our specification is sufficiently general to allow $x^* = \underline{x}$, in which case $J(x) \geq x$ for all $x \geq \underline{x}$.[7]

It is assumed that the manager's control benefits are higher under C than under R. Indeed, as a normalization, it is assumed that $B = 0$ under R. We may think of R as a strategy that can only be implemented by a competing manager, in which case the the necessary control transfer is achieved via a takeover. Alternatively, R may be an operating policy that is optimal relative to C to the extent that the firm is sufficiently successful in its early stages, and that the changes in the firm's operating policy associated with R reduce the ability of the manager to capture control benefits. In this sense, R represents the firm's growth potential, with a higher value of $J(x)$ at each x corresponding to a larger growth potential (or more growth opportunities).

3 The Optimal Contract

Assume that x is verifiable. This makes it possible to characterize the optimal policy (or contract) without having to think about how to implement it. Section 4 assumes that x is unverifiable and shows how the optimal contract can be implemented via the firm's financial structure.

The date 1 optimal operating policy is a function of x. Let x_L (x_R) denote the critical x below (above) which it is optimal to choose L (R) over C. Note that if $x_L > x_R$, then C will never be optimal. To avoid this, I will focus on the case for which the optimal solution implies $x_L < x_R$, which implies that C is optimal for $x \in [x_L, x_R]$.

The manager receives his compensation in form of expected control benefits $E(B) \equiv B \int_{x_L}^{x_R} f(x)dx$ and a fixed wage w.[8] It is assumed that the utility of the manager is linear in wealth and private benefits. His expected utility is therefore $E(U) = w + E(B)$.

Let $\overline{U}$ denote the reservation utility of the manager. To ensure that $w \geq 0$, let $\overline{U} \geq B$. And to ensure that w is riskless, let $w \leq x_L$. (However, since the manager is risk neutral any complications that arise from letting w be risky are purely notational).

[7] The expected *return* from choosing R instead of C is given by $r \equiv J(x)/x$. For example, putting $J(x) = ax - c$, where $a > 1$ and $c \geq 0$, we have that $r = a - c/x$, so that the expected rate of return from R is constant for $c = 0$ (in which case $x^* = \underline{x}$), and increasing in x for $c > 0$ (or, for $x^* > \underline{x}$).

[8] The effect of giving the manager a positive ownership stake in the firm is discussed below.

Given (x_L, x_R), the date 0 value of the firm is

$$v \equiv \int_{\underline{x}}^{x_L} lf(x)dx + \int_{x_L}^{x_R} xf(x)dx + \int_{x_R}^{\overline{x}} J(x)f(x)dx. \tag{1}$$

The optimal contract is the solution to the following maximization problem:

$$\max_{x_L, x_R} v - w \tag{2}$$

subject to

$$w + E(B) \geq \overline{U}, \tag{3}$$

where (3) represents the manager's participation constraint. Competition in the labor market ensures that (3) will be satisfied with equality. Substituting (3) into (2) yields the following unconstrained problem:

$$\max_{x_L, x_R} v + E(B) - \overline{U} \tag{4}$$

The first order conditions for this problem are:

$$l = x_L + B \tag{5}$$

and

$$J(x_R) = x_R + B, \tag{6}$$

where the second order conditions for maximum are easily shown to be satisfied. The fact that $B > 0$, creates a wedge between the optimal contract, given by (x_L, x_R), and value maximization, given by (l, x^*). Specifically, since $B > 0$ and $J'(x) > 1$, it is the case that $x_L < l$ and $x_R > x^*$. (See Figure 1).

It has so far been assumed that the manager receives his direct compensation in form of a fixed wage. To what extent will his incentives change if he instead is given a positive ownership stake $\alpha \in (0, 1)$ in the firm? The manager on date 1 will then choose L over C if $\alpha l \geq \alpha x + B$, or $l \geq x + B/\alpha$, and similarly R over C if $J(x) \geq x + B/\alpha$. Hence, although the incentives of the manager and investors become better aligned by the manager holding a positive ownership stake in the firm, he will implement the optimal policy if (and only if) he owns 100% of the firm. As will become clear, also the optimal financial structure of the firm will be unaffected by whether the manager holds a positive ownership stake or not in the firm.

4 Implementing the Optimal Contract

In Section 3 we characterized the optimal contract under the assumption that x is verifiable. We now let x be unverifiable and show how the optimal contract can be implemented using the firm's financial structure.

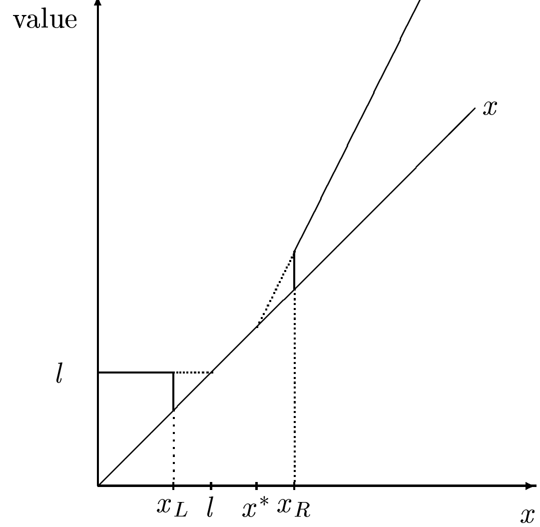

Figure 1: The figure depicts the date 1 value of the firm as a function of x and the optimal policy (for linear $J(\cdot)$).

Recall that the optimal policy calls for L if $x \in [\underline{x}, x_L)$. The desired control transfer in this case can be achieved issuing short-term debt with face value $D_s = x_L$, to be paid on date 1. Since the firm generates no date 1 cash flow, this claim is refinanced in full on this date, which is done by issuing a new senior claim. Let $D_{s1}(i(x)); i(x) \in \{x, J(x)\}$ denote the face value of this claim, due on date 2. $D_{s1}(i(x))$ will be determined from

$$\int_{\underline{\omega}}^{D_{s1}-i(x)} (i(x)+\omega)g(\omega)d\omega + D_{s1}(1 - G(D_{s1} - i(x))) = D_s, \tag{7}$$

which implies that $D_{s1}(\cdot)$ raises just enough cash to pay off the claim D_s.

Proposition 1 describes the financial structure needed to implement the optimal contract.[9]

Proposition 1 *The optimal contract from Section 3 can be implemented by issuing a financial structure consisting of short-term debt with face value $D_s = x_L$ (due on date 1), (zero-coupon) junior long-term debt with face value $D_l = J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$, and voting equity with payoff* $\max[i(x) + \omega - J(x_R) - \overline{\omega}, 0]$. *These claims are held by separate investors, and the payment D_s due on the interim date is funded (unless the firm is liquidated) by issuing a senior debt claim with face value D_{s1} determined from (7). The long-term debt claim permits just enough dilution on date 1 to enable the firm to refund its initial short-term claim D_s for all $x \geq x_L$. Dilution in excess of this is avoided by attaching a covenant to the long-term debt claim that specifically restricts the amount of money raised on date 1 not to exceed D_s.*

The role of financial structure in the present setting is to transfer control to the firm's security holders on date 1 when either $x < x_L$ or $x > x_R$, and hence protect the manager's

[9]All proofs in the Appendix.

control for $x \in [x_L, x_R]$. To avoid default, the firm must be able to raise at least $D_s = x_L$ on the interim date. And to be able to do so for all $x \in [l, x_L]$, the short-term claim issued must be senior to the existing long-term claim. As a result, long-term debt must allow some dilution of their claim on date 1. However, since it will be in the interests of shareholders to raise an amount in excess of D_s (and take it out as a date 1 dividend), the long-term claim must carry a covenant that restricts the amount of debt that can be issued on date 1 not to exceed D_s. In other words, the firm issues on the initial date a short-term senior debt claim with face value $D_s = x_L$, along with a junior long-term debt claim that permits just enough dilution on the interim date to allow the initial short-term claim to be refunded for any $x \geq x_L$.[10]

To enable shareholders to force the manager to undertake R (or to replace the manager altogether), outside equity must have formal rights to do so. Ideally, shareholder control should be contingent on the event 'x exceeds x_R' but since x in unverifiable this is not possible. Instead, outside equity must be given either unconditional control, or no control. But while no control means that R never will be implemented, unconditional control implies that R will be implemented whenever this is consistent with firm value maximization.[11] However, shareholder control can be made contingent as desired by the firm issuing a long-term debt claim with face value $D_l = J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$. This claim creates a sufficient debt overhang on date 1 to ensure that shareholders will exercise their control rights and implement R if and only if $x > x_R$.[12]

Note that it is important here that long-term debt and outside equity are held by different investors.[13] Otherwise, the presence of long-term debt would not represent a debt overhang, and shareholders would implement R for any $x > x^*$. Also, an implicit assumption that has been made is that investors refrain from renegotiating the contract ex post. For example, shareholders may on date 1 purchase the firm's long-term debt and thereby get rid of the debt overhang. However, if long-term debt is dispersely held, then hold out problems may make this very costly. Rather than adding this bit of complexity, it is assumed that investors simply

[10] To see this, suppose that $x = x_L$ so that the founder must pledge 100% of the date 2 cash flow in order to avoid default. Since pledging 100 % of the future cash flow can be done only if the claim issued is senior to existing claims, it follows that the optimal liquidation policy can be implemented if and only if debt issued on the interim date is senior to existing claims. This follows an argument laid out by Diamond (1991) for a somewhat different setting (see Introduction).

[11] Indeed, with unconditional control to outside equity the optimal policy will be abandoned altogether and the firm will be liquidated or restructured whenever this is consistent with value maximization.

[12] Note that since control transfers are costless in the present model, D_l must be set so that shareholder obtain a zero pay-off (with probability one) whenever $x \in [\underline{x}, x_R]$, in which interval shareholders weakly prefer not to exercise their control rights. Similarly, they will strictly prefer to exercise their control rights and intervene for $x > x_R$.

[13] For other papers that propose a role for multiple outside investors holding diverse claims on the firm see e.g. Dewatripont and Tirole (1994) and Berglöf and von Thadden (1994).

refrain from attempting such ex post renegotiation.[14]

5 Empirical Content

We now relate the firm's financial structure to changes in managerial control benefits, liquidation value, going concern value, and growth opportunities.

An increase in the manager's control benefits B implies a substitution away from value maximization, leading in turn to a decrease in x_L and to an increase in x_R. This has the following effect on the firm's financial structure.

Lemma 1 *An increase in the manager's private benefits B leads to less short-term debt D_s, more long-term debt D_l, and more total debt D.*

According to the free cash flow theory of Jensen (1986), a high level of debt has the effect of reducing the ability of the manager to extract rents at the expense of shareholders. Lemma 1 suggests that a high level of debt level may indicate the presence of high control benefits, so long as this is observed in combination with a high proportion of long-term debt.

Lemma 2 *An increase in the firm's liquidation value l leads to more short-term debt D_s, less long-term debt D_l, and less total debt D.*

An increase in l will decrease the critical value of x below which L will be implemented, and hence increase the amount of short-term debt. It will also decrease the amount of long-term debt that must be issued, since a higher amount of short-term debt issued initially will increase the amount of debt that must be issued on the interim date.[15]

Alderson and Betker (1995) define liquidation costs as the difference between the 'going concern' value of the firm and its liquidation value. To study the relationship between debt structure and liquidation costs in the present setting, suppose that the expected cash flow given x is $x(1+g)$ and $J(x)(1+g)$ under C and R. Hence, an increase in g may be interpreted as an increase in the going concern value of the firm.

Lemma 3 *An increase in the going concern value of the firm has no effect on the amount of short-term debt D_s, but decreases (increases) the amount of long-term debt D_l if $J(x_R) < (>) \frac{B}{1+g}\frac{J'(x_R)}{J'(x_R)-1}$.*

[14] It should be noted that this is a common (often implicit) assumption in the literature; see e.g. Israel (1991), Dewatripont and Tirole (1994), and Berkovitch and Israel (1996).

[15] It may be interesting to note that the result of Lemma 2 is identical to that derived by Park (2000) in a different setting.

All else constant, an increase in $J(x)(1+g)$ will require an increase in the amount long-tem debt D_l. But an increase in g will also decrease x_R and hence decrease D_l, so that whether D_l will be increasing or decreasing in g will depend on these two effects.

Discussion. While the prediction of Lemma 2 is unambigous, the empirical evidence on debt structure and liquidation value is mixed. For example, Titman and Wessels (1988) find an insignificant positive relation with respect to both short-term and long-term debt. Kim and Sorensen (1986) find a positive relation between liquidation value and leverage, while Kale, Noe, and Ramirez (1991) find evidence of a negative (as well as positive) relation.[16]

Lemma 2 and 3 combined, however, predict that greater liquidation costs will lead to less short-term, less long-term debt, and (depending on parameter values) an increase in the proportion of long-term debt. This result is consistent with the empirical evidence of Alderson and Betker (1995) that higher liquidation costs are associated with less short-term, less long-term debt, and a higher proportion of 'public unsecured debt'. Titman and Wessels (1988) observe that "the predicted effects were not uncovered because the indicators used [...] do not adequately reflect the nature of the attributes suggested by theory." In this respect, Lemmas 2 and 3 suggest that the way one measures and defines liquidation costs (or values) may have ambigous implications not just empirically but also theoretically.

The next lemma relates the firm's financial structure to the presence of growth options (as measured by a greater $J(x)$ for each x).

Lemma 4 *An increase in the firm's growth opportunities leaves the amount of short-term debt unaffected, while decreasing the amount of long-term debt and hence the total amount of debt.*

In Myers (1977), the presence of long-term debt creates a debt overhang that may leads firms to pass up valuable investments. The presence of greater growth opportunities, therefore, should lead to less long-term debt. In the present paper, greater growth opportunities increases the importance of firm value relative to control benefits, which in turn decreases the need for long-term debt as a protection to the manager.

Consistent with Lemma 4, Barclay and Smith (1994) find a significant negative relation between debt maturity and growth opportunities, and Smith and Watts (1992) (among others) find a negative relation between growth opportunities and total debt level. Also Stohs and Mauer (1995) uncover a negative relation between total debt level and growth opportunities, but find an insigificant (positive) relation between debt maturity and growth opportunities, suspecting that "the Barclay and Smith regressions are misspecified because they do not control for leverage."

[16] See review article by Harris and Raviv (1991), and see Alderson and Betker (1995) for a more recent discussion.

Lemma 5 *An increase in the firm's growth opportunities will lead to an increase in the manager's cash compensation w (and to a corresponding reduction in his expected control benefits).*

This result arises from a simple substitution along the manager's participation constraint, and is consistent with Gaver and Gaver's (1993) finding that higher growth firms pay higher cash compensation to their managers than lower growth firms.

6 Concluding Remarks

This paper develops a simple model in which the firm's financial structure serves the purpose of optimally allocating control and cash flows between the firm's manager and its investors. The optimal financial structure is shown to consist of short-term senior debt, long-term junior covenanted debt, and outside equity with unconditional control.

It is well known that the choice of financial structure is irrelevant in frictionless markets (Modigliani and Miller [1958]). In the present setting, the particular frictions that give rise financial structure relevance include private control benefis and non-verifiable interim information. The main contribution of the paper is to show that the type of complex financial structures observed in practice where both capital structure (debt and equity) and debt structure (priority and maturity) play non-trivial roles can be generated in a simple setting based on these frictions.

Appendix

Proof of Proposition 1

(i) $x \in [\underline{x}, x_L)$. Then $E(x+\omega) = x < D_s$ so that under C the manager will be unable to raise enough cash to satisfy date 0 short-term lenders. Date 0 short term lenders therefore will take control, liquidate the firm, and hence receive $\min(D_s, l) = D_s = x_L$ (while long-term lenders receive $l - x_L$).

(ii) $x \in [x_L, x_R]$. In this case we must show that (a) the firm is able to raise enough cash to satisfy date 0 short-term lenders, (b) R will not be profitable, and (c) shareholders will refrain from forcing the firm to be liquidated (which is profitable for the firm's security holders for $x \in (x_L, l)$).

Condition (a) follows since the new claim is senior to existing claims and since $E(x+\omega) = x > D_s = x_L$ for all $x \in [x_L, x_R]$.

Consider then condition (b). The date 1 value of equity under R is given by

$$E\{\max[J(x) + \omega - D_l - D_{s1}, 0]\} = 0. \qquad (A.1)$$

Insert the expression for $D_l = J(x_R) + \overline{\omega} - D_{s1}$ into $(A.1)$ to find that the date 1 equity value is

$$E\{\max[(J(x) - J(x_R)) + (\omega - \overline{\omega}), 0]\}, \qquad (A.2)$$

which, since $J(x) \leq J(x_R)$ and $\omega \leq \overline{\omega}$, is zero for all $x \in [x_L, x_R]$.

Consider finally condition (c). The face value D_l of long-term debt is determined so that

$$J(x_R) = V_l(J(x_R), D_l(J(x_R))) + D_s, \qquad (A.3)$$

where $V_l(\cdot,\cdot)$ is the date 1 value of a long-term debt claim with face value $D_l(\cdot)$. D_s is similarly the date 1 value of a debt claim issued on date 1 with face value D_{s1} due on date 2.

Let $V_e(l)$ denote the cash received by shareholders if the firm is liquidated on date 1. We want to prove that $V_e(l) = 0$. Suppose to the contrary that $V_e(l) > 0$. If the firm is liquidated, its liquidation proceeds l will be distributed to claimholders according to stated priority rules. By the assumption that $V_e(l) > 0$, this implies that long-term lenders receive D_l and that short-term lenders receive D_s. $V_e(l)$ is therefore the residual value determined from

$$l = V_e(l) + D_l + D_s. \qquad (A.4)$$

The assumption that $V_e(l) > 0$ implies now that

$$V_e(l) = l - D_l - D_s > J(x_R) - V_l(\cdot;\cdot) - D_s = 0, \qquad (A.5)$$

or that

$$l - D_l > J(x_R) - V_l(\cdot;\cdot); \tag{A.6}$$

which, by the fact that $V_l(\cdot;\cdot) < D_l$, implies that $l > J(x_R)$, which, by the assumption that $x_R > x_L$, must be incorrect. This implies a contradiction and hence that $V_e(l) > 0$ is wrong. We may therefore conclude that $V_e(l) = 0$. □

Proof of Lemma 1

The fact that D_s is decreasing in B is seen directly from $D_s = x_L = l - B$. The fact that D_l is increasing in B can be seen from the expression $D_l = J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$ and the first order condition $J(x_R) = x_R + B$; an increase in B leads to an increase in x_R (since $J'(x) > 1$), which in turn leads to a greater $J(x_R)$, a lower $D_{s1}(J(x_R))$ and thus higher $D_{s1}(\cdot)$.

To see that $D = D_s + D_l$ is increasing in B, note that $D = l - B + J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$. Taking the total derivative of D with respect to B yields:

$$\frac{dD}{dB} = -1 + J'(x_R)[1 - D'_{s1}(J(x_R))]\frac{dx_R}{dB}$$

$$= -1 + J'(x_R)[1 - D'_{s1}(J(x_R))]\frac{1}{[J'(x_R) - 1]} \tag{A.7}$$

so that $\frac{dD}{dB} > 0$ if

$$J'(x_R)[1 - D'_{s1}(J(x_R))] \geq [J'(x_R) - 1], \tag{A.8}$$

which is so since $D'_{s1}(J(x_R)) < 0$ by (7). □

Proof of Lemma 2

The fact that D_s is increasing in l is seen directly from $D_s = x_L = l - B$. To see that $D_l = J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$ recall first that $D_{s1}(i(x))$ is determined by (4), or

$$\int_{\underline{\omega}}^{D_{s1} - i(x)} (i(x) + \omega) g(\omega) d\omega + D_{s1}(1 - G(D_{s1} - i(x))) = D_s = l - B, \tag{A.9}$$

from which it can be observed that a greater l leads to an increase in D_{s1}. The larger D_{s1} can in turn be seen from $D_l = J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$ to decrease D_l.

Finally, to see that $D = D_l + D_s$ is decreasing in l, differentiate D with respect to l:

$$\frac{dD}{dl} = 1 - \frac{dD_{s1}}{dl} = 1 - \frac{1}{1 - G(D_{s1} - J(x_R))} < 0. \tag{A.10}$$

□

Proof of Lemma 3

The first order conditions are now

$$l = x_L(1 + g) + B \tag{5'}$$

and

$$J(x_R)(1+g) = x_R(1+g) + B. \tag{6'}$$

The amount of short-term is then $D_s = (1+g)x_L$ and the amount of long-term debt level is $D_l = (1+g)J(x_R) + \overline{\omega} - D_{s1}((1+g)J(x_R))$. To see that D_s is unrelated to D_s, note by (5') that $(1+g)x_L = l - B$, so that $D_s = l - B$. Taking the change in D_l with respect to g yields

$$\frac{dD_l}{dg} = \Big[J(x_R) + (1+g)J'(x_R)\frac{dx_R}{dg}\Big]\Big(1 - D'_{s1}(J(x_R))\Big). \tag{A.11}$$

Since $D'_{s1}(\cdot) < 0$, $\text{sign}\frac{dD_l}{dg} = \text{sign}[J(x_R) + (1+g)J'(x_R)\frac{dx_R}{dg}]$. Using the fact that $\frac{dx_R}{dg} = -\frac{B}{(1+g)^2}\frac{1}{J'(x_R)-1}$, we obtain the inequality as stated in the lemma. □

Proof of Lemma 4

Although there are different ways of doing this, assume for simplicity that $J(x)$ is linear (as in figure 1). Specifically, assume that $J(x) = ax - c$ with $a > 1$ and $c \geq 0$ (as in footnote 6). In this case, it is straight forward to show that $x^* = \frac{c}{a-1}$ and $x_R = \frac{B+c}{a-1}$, so that $x_R > x^*$ so long as $B > 0$. Now, given this expression for x_R, it will be the case that

$$J(x_R) = \frac{aB+c}{a-1}. \tag{A.12}$$

By Proposition 1, we know that $D_l = J(x_R) + \overline{\omega} - D_{s1}(J(x_R))$. Taking the change in D_l with respect to a yields

$$\begin{aligned}\frac{dD_l}{da} &= \frac{dJ(x_R)}{da} - \frac{dD_{s1}(\cdot)}{dJ(x_R)}\frac{dJ(x_R)}{da} \\ &= -\frac{(B+a)}{(a-1)^2}\Big[1 - \frac{dD_{s1}(\cdot)}{dJ(x_R)}\Big].\end{aligned} \tag{A.13}$$

Hence, since $B + c > 0$ and $\frac{dD_{s1}(\cdot)}{dJ(x_R)} < 0$, it follows that $\frac{dD_l}{da} < 0$, which means that D_l is decreasing in the firm's growth potential. Since the firm's short-term debt level $D_s = x_L = l + B$ is clearly unrelated to a, this implies that the firm's total debt level of $D = D_s + D_l$ must be decreasing in a (and hence decreasing in the firm's growth opportunities). □

Proof of Lemma 5

From the first order condition (6), since $J'(x) < 1$, it follows that an increase in $J(x)$ will lead to a decrease in x_R. In turn this will lead to a decrease in expected control benefits $E(B)$ and hence to an increase in w. The latter follows directly from the manager's participation constraint (3). □

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